**NONPARAMETRIC ESTIMATION OF THE SPATIAL CONNECTIVITY
MATRIX BY THE METHOD OF MOMENTS USING SPATIAL PANEL DATA**

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We use moments from the covariance matrix for spatial panel data to estimate the parameters of the SAR model, including the spatial connectivity matrix W . In the unrestricted SAR model the parameters are exactly identified by the moments in the covariance matrix. The restricted SAR model is over-identified, but we suggest that its parameters may be estimated by GMM and its restrictions may be tested empirically. If Gini covariances are used instead of conventional (Pearson) covariances, the estimated W matrix is asymmetric. We also suggest that the reduced rank covariance matrix will estimate W and the SAR parameters more efficiently.

1. Introduction

Ever since its inception in the 1970s the spatial connectivity matrix, commonly denoted by W , has been imposed rather than estimated. In principle, goodness-of-fit tests may be used to choose between rival definitions of W . In practice, however, most researchers impose W without testing its restrictions empirically. In this paper we propose a methodology to estimate W from spatial panel data. Our estimator is based on the method of moments and is entirely nonparametric. Specifically, our estimator is designed for the spatial autoregression model (SAR) in which W is estimated rather than imposed, and in which the SAR model is heterogeneous across spatial units.

Unlike temporal connectivity, which is unidirectional, spatial connectivity is multidirectional. Temporal connectivity moves forward from $t - 1$ to t . Time series would be much more complicated if time also ran backwards from t to $t - 1$. Spatial connectivity not only runs forwards and backwards, it also runs upwards and downwards. Whereas temporal connectivity in the ARMA model is straightforward and unambiguous, spatial connectivity in the SARMA model is ambiguous since it is unclear how W should be specified.

Spatial models assume a particular type of spatial dependence. This can be based in Euclidean, social or economic distance. As models are very sensitive to the type of spatial dependence assumed, the choice of weight matrix becomes a critical issue in model specification. The literature on constructing W suggests two principle approaches. The first relates to imposing W on the data. The second relates to inferring W from the data. The various metrics available for imposing W have been reviewed in Getis and Aldstadt (2004) and guidelines for constructing weight matrices have been outlined by Griffith (1996).

Whichever approach is taken, a key issue in defining W relates to the extent to which spatial connectivity is represented. A weight matrix that imposes a spatial structure on the data does not necessarily correspond to the reality of spatial relationships in the data. Ideally, the choice of W should be grounded in theory and the choice of spatial structure should be based on

a defensible notion of how space affects interaction. In practice, data availability and convenience often inform the choice. In the absence of any theoretical dependence structure, a model using an inappropriate specification of W will be misspecified (Getis and Aldstadt 2004).

The 'appropriate' W is usually derived in one of three ways summarized in Table 1. As noted, W can be imposed exogenously based on a general notion as to how distance affects connectivity. This approach is broadly theoretical and is driven by a general gravity-type notion of how proximity affects interaction. Any metric of distance decay can be used such as inverse distance raised to a power (Getis and Aldstadt 2004) or bandwidth distance decline (Fotheringham, Brunson, and Charlton 2002). A composite W matrix can also be constructed. Shi, Zhang and Liu (2004) that combines distance (geographic space) with indicators of interaction (attribute space) such as population size across spatial units.

A second method that also imposes W on the data, uses indicators based on contiguity and contagion (Rey and Montouri 1999). The geometry of spatial units is important with the resultant W matrix reflecting the configuration of the cells and the contiguity or distance between them. The nature of W is grounded in Tobler's law 'first law of geography' to the effect that 'nearness matters'. The resulting data-driven matrix is predictive in that it assumes a connectivity structure based on the fact that closer neighbors have more effect than distant ones.

The third approach is to infer W from the data itself using geostatistical modeling. This is the approach adopted here and in other recent studies (Aldstadt and Getis 2006, Bhattacharjee and Jensen-Butler 2006, Fernandez Vazquez et al 2007, Kankamu 2005, Oud and Folmer 2008, Paelink 2007). In this respect, W describes the spatial relationships inherent in the data. Once spatial weights have been estimated, they can then be interpreted in terms of gravity, contiguity and other concepts that have been used to design W ex ante.

Three main approaches have been suggested for estimating W . The first is data-driven. For example, Getis and Aldstadt (2006) propose an algorithm which searches for spatial clustering in the vicinity of selected seeds. The algorithm which is based on the local statistics model in Getis and Aldstadt (2004) essentially constructs a data-driven empirical representation of W . In an empirical application they report that the t -statistic on the spatial lag coefficient is only 1.63 when W is based on contiguity but it is 98.79 when W is derived through their

algorithm. Critics may argue however, that because their algorithm mines the data, the goodness-of-fit was bound to be superior to its rivals.¹

Table 1 Review of W

Metric of Association	Nature of W	Role of Matrix	Approach
Distance-decay or proximity-based <ul style="list-style-type: none"> • $(1/d^2)$ • Bandwidth distance decline • social interaction indicators (pop size, commuting) 	Imposed	Explanatory-predictive	Broadly theoretical
Geometric or contiguity-based <ul style="list-style-type: none"> • Rook/Queen • n^{th} nearest neighbor distance • Lengths of Shared Borders, Perimeters 	Simple application of Tobler's first (nearest neighbor) rule	Explanatory-predictive	Mechanical-data driven
Geostatistical models <ul style="list-style-type: none"> • AMOEBA • Spatial filtering • Maximum entropy • Autocovariance matrix derived from spatial error model • Bayesian estimation • Latent variable modeling 	Estimated (spatial association within the data)	Descriptive-Estimation limited by nature of data	Theoretical-model driven

The second is the eigenvector spatial filtering methodology originally suggested by Griffith (1996). The principle eigenvector of W provides a measure of the relative positioning of each spatial unit and expresses the general degree of connectivity between spatial units. Other eigenvectors, or spatial filters, express different aspects of spatial connectivity, including regional basins, center – periphery, etc. Several authors have tested which spatial filters are

¹ Their SAC coefficient is 0.97 which implies the residuals contain a spatial unit root.

statistically significant (Getis and Griffith 2002, Tiefelsdorf and Griffith 2007, Dray, Legendre and Peres-Neto 2006). If all spatial filters are statistically significant then W is correctly specified. If not, they suggest using the significant filters only, which is equivalent to modifying the specification of W in light of the data. By using only the significant spatial filters and dropping the insignificant ones, one is essentially estimating the spatial structure semi-parametrically. The methodology is semi-parametric because W itself is parametric. Results obtained by this methodology naturally depend on how W was parameterized in the first place. If W was incorrectly specified, the spatial filters will be miss-specified too.

A third approach uses the covariance matrix from a spatial regression. Bhattacharjee and Jensen-Butler (2006) estimate a SAC model with autoregressive errors. They obtain spatial weights from the estimated covariance matrix of the spatial errors (i.e. from the observed pattern of spatial autocorrelation). They use this method to study the diffusion of housing demand across UK regions. Their results show significant heterogeneity in regional housing markets indicating that not just physical distance but also social and cultural distance between areas seems to be important.

Other recent attempts at inferring W include grouping correlation coefficients using an optimization algorithm so that observations that are correlated are also related spatially (Paelink 2007). Folmer and Oud (2008) propose using structural equation modeling (SEM) with observed and latent variables. In SEM, the structural part of a model represents that causal relations between the latent variables and the measurement part shows the relationship between the latent variables and their observable indicators. The latent variable approach replaces spatially lagged dependent variables by the latent variables in the structural model and models the relationship between spatially lagged variables and their observed indicators in the measurement model. Their study compares W with the latent variable estimators and shows the closeness of the estimates between them and the added flexibility of the latter. Kakamu (2005) uses Bayesian estimation to create a distance based W matrix that captures both the intensity of spatial interaction and the geometric pattern of decay. Using Markov Chain Monte Carlo methods to estimate model parameters he shows how the exponential pattern of distance decay can be used with both SAR and SAC models. Finally, the general maximum entropy (GME) model has also been used to empirically estimate W (Fernandez-Vazquez et al 2007). Invariably, in estimating a SAR model, insufficient data causes W to be imposed. The GME model

estimates the elements of W jointly with the residuals of the model parameters allowing for the estimation of unknown probability distributions in situations with limited data. Rather than impose a spatial structure for W , the GME approach requires the researcher to define plausible values for the model's parameters.

We propose a methodology, based on the method of moments, in which the spatial moments generated by the data are used to estimate W nonparametrically. Specifically, we hypothesize a SAR model to be estimated from spatial panel data. We infer W directly from the covariance matrix for the data, from which we also infer heterogeneous SAR coefficients. Since the covariance matrix is symmetric the estimate of W is necessarily symmetric. However, we suggest that asymmetric W may be estimated using the Gini covariance matrix of the data. We also suggest that more efficient estimates of W and related parameters may be estimated if the rank of the covariance matrix of the data is less than full.

While all the parameters of the SAR model are identified by the moment conditions, the same does not apply to the SARMA model, which is under-identified because there are more parameters than moments. On the other hand, the parameters of restricted SAR models are over-identified because there are more moments than parameters. To test such restrictions we suggest a GMM – type estimator for the restricted model. We use spatial panel data for regional house prices in Israel to illustrate the application of the methodology.

2 Methodology

2.1 Basic Method

Let y_t denote a column vector of outcomes in time period $t = 1, 2, \dots, T$ in N spatial units where $i = 1, 2, \dots, N$. The panel SAR model may be written as:

$$y_t = \alpha + BWy_t + \varepsilon_t \quad (1)$$

where α is an $N \times 1$ vector of constants and B is a diagonal matrix with diagonal elements β_i . If the SAR coefficients are homogeneous so that $\beta_i = \beta$, B is replaced by β in equation (1). The variance covariance matrix $\Sigma = E(\varepsilon\varepsilon')$ is assumed to be time invariant (temporal homoscedasticity), diagonal (no spatial autocorrelation between ε_i and ε_j) but is spatially

heteroscedastic so that the variance of ε (σ^2_i) may vary between spatial units. The spatial Wold representation of equation (1) is:

$$y_t = A(\alpha + \varepsilon_t) \quad (2)$$

$$A = (I_N - BW)^{-1}$$

Let $V = yy'$ denote the covariance matrix of the y 's estimated from spatial panel data.

Substituting equation (2) for y gives:

$$V = A\Sigma A' = H \quad (3)$$

Since V is symmetric it contains $\frac{1}{2}N(N+1)$ independent elements. Since $w_{ii} = 0$, W is symmetric and $\sum_j w_{ij} = 1$ there are $\frac{1}{2}N(N-1)$ unknown w_{ij} coefficients. There are N unknown SAR coefficients and N unknown variances (diagonal components of Σ), making $\frac{1}{2}N(N+1)$ unknown parameters altogether. Therefore equation (3) may be used to solve these unknown parameters. It may be shown that each element of H contains all the parameters of the SAR model including the elements of W , B and Σ . Moreover, these elements are quartics because A is quadratic. Therefore the solutions to equations (3) are nonlinear in the unknown parameters and in general there are multiple solutions because of their quartic structure.

Because equation (3) is nonlinear in the unknown parameters we compute them numerically². We equate each independent element of V with its counterpart in H , i.e. we equate v_{12} with h_{12} etc. Recall that $v_{21} = v_{12}$ and h_{21} is equal to h_{12} because of symmetry. We rule out solutions with negative variances, so that in practice the solution seems to be unique. In short, we use V to estimate the N SAR model parameters (B), the $\frac{1}{2}N(N-1)$ elements of W , and N elements of Σ . Equation (3) does not directly estimate α but this is easily obtained since from equation (2) $\hat{\alpha} = \hat{A}^{-1}\bar{y}$. The variances of the estimates of W and B may be obtained by panel bootstrapping V . Details are provided below.

Because the parameters of the SAR model are exactly identified, the parameters of the SARMA model are under-identified. In the SARMA model $e_t = CWe_t + \varepsilon_t$ where C is a diagonal matrix with N moving-average parameters on the diagonal. In this case equation (3) applies with A now equal to:

² In fact we use the inverse of equation (3), which is solved in Matlab using the Gauss - Newton algorithm. Since in our case $N = 9$ this involves solving 45 equations for 45 unknown parameters for which the starting values include positive variances.

$$A = (I - BW)^{-1}(I - CW)^{-1} \quad (4)$$

The identification deficit is equal to N because C contains N unknown parameters to be estimated. This deficit may be reduced by arbitrarily restricting the number of independent SAR and SMA parameters to equal N.

2.2 Principal Components

We suggest that more robust estimates of the SAR model parameters may be obtained by applying equation (3) to the statistically significant principal components of V. These principal components contain information on spatial dependence in the data. If there are $K < N$ statistically significant principal components the rank of V is equal to K. The $p = N - K$ statistically insignificant principal components do not contain useful information on spatial dependence in the data. We use the eigenvalue test due to Schott (2006) to determine p:

$$S = \frac{pT}{2} \left[\frac{p^{-1} \sum_{i=1+K}^N \lambda_i^2}{[p^{-1} \sum_{i=1+K}^N \lambda_i]^2} - 1 \right] \approx \chi^2_{\frac{p(1+p)}{2}-1} \quad (5)$$

If S is smaller than the critical value of chi square we cannot reject the hypothesis that the p smallest eigenvalues are not significantly different from the K'th.

The spectral decomposition of V is $V = G\Lambda G'$ where Λ is the NxN eigenvalue matrix of V and G is the NxN matrix of its eigenvectors. The Nx1 principal component vector is equal to $P = G'y$. Let Λ^* denote the KxK eigenvalue matrix formed by the statistically significant principal components, let G^* denote its NxK matrix of eigenvectors, and let $P^* = G'^*y$ be the Kx1 vector of significant principal components. Finally, we denote $y^* = LP^*$ where L is an NxK matrix of loadings obtained³ using the generalized inverse of G'^* : $L = G'^*{}^+ = (G^*G'^*)^{-1}G'^*$. The variance covariance matrix formed by the statistically significant principal components is $V^* = y^*y'^*$. Using V^* rather than V eliminates statistically insignificant spatial dependence in the data, thereby increasing the robustness of the estimates of the SAR model.

2.3 Asymmetric W

³ We used the PCA procedure in Matlab.

Since V is symmetric the estimate of W is naturally symmetric. We suggest that asymmetric estimates of W may be obtained by using the Gini covariance matrix, V_G , which uses ranked data. Let R_{it} denote the rank of Y_{it} out of T such that $R = 1$ for the smallest value and $R = T$ for the largest. The Gini covariance between i and j is defined as $\text{cov}(Y_i R_j)$. Since $\text{cov}(Y_i R_j)$ is generally different⁴ from $\text{cov}(Y_j R_i)$ the Gini covariance matrix V_G is not symmetric. In this case W will not be symmetric. Normalizing the sum of weights to unity W has $N^2 - 2N$ independent elements, which together with the $2N$ parameters of B and Σ gives N^2 unknown parameters, which may be solved using equation (3) from the N^2 elements of V_G .

2.4 Testing Restrictions

The SAR parameters and W are exactly identified because the number of moments is equal to the number of parameters to be estimated. Suppose, however, that the null hypothesis is that the SAR coefficients are homogeneous so that H_0 is $\beta_i = \beta$. Therefore there are $N - 1$ fewer parameters to estimate in the restricted model, and the moments over-identify β and W . The generalized method of moments (GMM) is designed for over-identified situations and chooses estimates of β and W that minimize the violations of the moment conditions. Suppose $N - 1$ moments are excluded, and the restricted parameters are estimated from the retained moments. The retained moments will hold exactly but the excluded moments will be violated. The violation for the j 'th excluded moment is the moment in the data (v_j) minus its predicted value q_j . Let $Q(\beta, W)$ denote the metric of violations for the $N - 1$ excluded moments. We repeatedly exclude $N - 1$ moments with replacement and compute Q for each trial. In the r 'th trial:

$$Q_r = \sum_{j=1}^{N-1} \omega_j q_{rj}^2$$

The GMM estimator for β and W is defined when Q is minimized. To test the restricted SAR model we compare $\text{trace}(\Sigma)$ from the restricted model to its unrestricted counterpart, where $T[\text{trace}\Sigma_R - \text{trace}\Sigma_{UR}] \sim \chi^2_{N-1}$. If the calculated value exceeds the critical value the restrictions are rejected.

⁴ Unless Y_i and Y_j are “exchangeable”, which arises when the marginal distributions of Y_i and Y_j are identical.

3 Empirical Application

3.1 The Data

To illustrate the methodology we use spatial panel data for the logarithm of regional house prices (measured in constant prices) in Israel observed annually between 1975 and 2006 for 9 regions (see map). See Figure 1. The panel unit root test statistic (IPS) due to Im, Pesaran and Shin (2003) is -1.74 and its common factor counterpart (CIPS) due to Pesaran (2006) is -1.902. Therefore the data are clearly nonstationary⁵. However, the data are stationary in first differences since the IPS and CIPS statistics are -6.029 and -4.216 respectively. We therefore report results for the log levels and log differences in house prices.

The correlation matrices for the levels ($d = 0$) and first differences ($d = 1$) of the log of real house prices are given in Table 2.

Table 2 Correlation Matrix for Regional House Prices

	d	South	Dan	Tel Aviv	Sharon	Center	Haifa	North	Jerusalem
Krayot	0	0.959	0.978	0.967	0.977	0.953	0.733	0.770	0.956
	1	0.711	0.888	0.797	0.865	0.755	0.824	0.225	0.748
South	0		0.954	0.943	0.953	0.959	0.709	0.918	0.028
	1		0.797	0.751	0.800	0.806	0.703	0.670	0.698
Dan	0			0.991	0.994	0.972	0.638	0.793	0.971
	1			0.950	0.947	0.891	0.817	0.391	0.907
Tel Aviv	0				0.987	0.961	0.648	0.774	0.980
	1				0.914	0.886	0.715	0.413	0.940
Sharon	0					0.971	0.662	0.779	0.975
	1					0.855	0.910	0.395	0.888
Center	0						0.667	0.765	0.948
	1						0.638	0.549	0.802
Haifa	0							0.469	0.708
	1							0.322	0.730
North	0								0.737
	1								0.326

3.2 Basic Results

In this section we report estimates of W and the SAR coefficients using equation (3) for the covariance matrix of regional house prices both in levels and first differences.

⁵ These unit root tests ignore spatial dependence. However, provided the SAC coefficient is not too large the size of these tests is not seriously affected. See Baltagi et al (2007).

The estimated W matrix is reported in Table 3 for the cases when V is estimated from levels ($d = 0$) and first differences ($d = 1$) of the data. The elements of the estimated W matrix range between 0.37 and -0.09 when $d = 0$ and between 0.41 and -0.17 when $d = 1$. Notice that some elements are negative suggesting negative spillovers rather than positive ones. Recall that the row sums of W are normalized to unity.

Table 3 Spatial Connectivity Matrix

	d	South	Dan	Tel Aviv	Sharon	Center	Haifa	North	Jerusalem
Krayot	0	0.167	0.257	0.105	0.201	0.020	0.203	0.068	-0.018
	1	0.174	0.216	0.094	0.213	0.207	0.375	-0.114	-0.166
South	0		0.006	0.107	0.020	0.367	0.143	0.200	-0.062
	1		0.055	-0.057	0.182	0.170	0.164	0.249	0.054
Dan	0			0.334	0.279	0.164	-0.092	0.013	0.018
	1			0.275	0.128	0.182	0.122	-0.012	0.090
Tel Aviv	0				0.094	0.052	-0.038	0.045	0.368
	1				0.133	0.183	-0.046	0.032	0.413
Sharon	0					0.175	-0.019	0.041	0.214
	1					0.131	0.157	0.002	0.169
Center	0						0.055	0.052	0.089
	1						-0.105	0.137	0.062
Haifa	0							0.108	0.356
	1							0.081	0.183
North	0								0.034
	1								-0.119

Table 4 reports the SAR coefficients and the variances of ε for each of the 9 regions. The SAR coefficients range between 0.42 and 1.1 when $d = 0$ and 0.26 and 1 when $d = 1$. Therefore there is substantial heterogeneity in the SAR coefficients, and some of the SAR coefficients indicate the presence of spatial unit roots⁶. On the whole regions with relatively large SAR coefficients when $d = 0$ also have relatively large SAR coefficients when $d = 1$.

The standard errors range between 0.025 and 0.3 when $d = 0$ and 0.03 to 0.35 when $d = 1$. Since the data are logarithms these standard errors are approximately percentages. The SAR model fits best in Dan and worst in the North.

Table 4 Estimates of SAR Coefficients and $\sigma(\varepsilon)$

⁶ Unit root tests for SAR coefficients may be found in Beenstock and Felsenstein (2009).

	d	Krayot	South	Dan	Tel Aviv	Sharon	Center	Haifa	North	Jerusalem
SAR	0	1.063	0.805	0.988	1.106	1.036	0.894	0.424	0.447	0.882
	1	0.753	0.719	0.943	1.020	1.016	0.850	0.735	0.262	0.749
$\sigma(\varepsilon)$	0	0.069	0.064	0.025	0.053	0.044	0.078	0.237	0.303	0.059
	1	0.065	0.070	0.030	0.044	0.061	0.072	0.120	0.346	0.055

We use the panel bootstrap to compute the standard errors of the estimated components of W and the SAR coefficients for the case when $d = 0$. This procedure draws samples from the residuals of the estimated SAR model (ε). Since these residuals are spatially uncorrelated we do not have to take direct account of spatial dependence in the bootstrap since this taken into consideration by the SAR model itself. We used 1000 replications⁷. Because the bootstrapped means differ slightly from the estimates reported in Tables 3 and 3, we report in Table 5 the means as well as the standard deviations of the bootstrapped parameters.

Table 5 indicates that the elements of W are estimated imprecisely. Even relatively large elements such as the connectivity coefficient between Sharon and Krayot (0.178), has a standard deviation of 0.1. Only 12 out of the 35 elements of W are statistically significant. A joint test easily rejects the hypothesis that $W = 0$.

Neither here nor elsewhere do we try to interpret the relative orders of magnitude of the estimated elements of W since our main purpose is to demonstrate the methodology. According to Table 4 the strongest spatial connectivity occurs between Jerusalem and Tel Aviv (0.372), between Haifa and Jerusalem and between Center and South (0.385). It is surprising, for example, that the connectivity between Tel Aviv and Center is not statistically significant whereas the opposite is true of the connectivity between North and South. This suggests that imposing W a priori in terms, for example, of distance and contiguity would have been quite inappropriate and misleading. Letting the data "speak for themselves" seems to lead to quite different estimates of W .

Table 5 Panel - Bootstrapped Estimates of W

		South	Dan	Tel Aviv	Sharon	Center	Haifa	North	Jerusalem
Krayot	W	0.152	0.276*	0.092	0.178	0.018	0.168*	0.124	-0.008
	sd	0.098	0.083	0.083	0.100	0.116	0.084	0.127	0.096

⁷ See Andrews and Buchinsky (2000) regarding the desirable number of bootstrap replications.

South	W sd		-0.019 0.114	0.135 0.111	0.007 0.119	0.385* 0.091	0.153* 0.066	0.201* 0.059	-0.062 0.115
Dan	W 1			0.353* 0.073	0.293* 0.076	0.166* 0.084	-0.081 0.043	-0.004 0.052	-0.011 0.084
Tel Aviv	W sd				0.082 0.103	0.043 0.075	-0.039 0.032	0.046 0.039	0.372* 0.065
Sharon	W sd					0.181* 0.078	-0.028 0.036	0.057 0.047	0.233* 0.112
Center	W sd						0.037 0.094	0.056 0.094	0.080 0.100
Haifa	W sd							0.132 0.341	0.385* 0.125
North	W sd								0.031 0.116

In Table 6 we report the bootstrapped SAR coefficients, which are similar to their counterparts in Table 4. The standard errors reported in Table 6 indicate that these SAR coefficients are, on the whole, measured with precision except in the North. Clearly all the SAR coefficients are statistically significantly different from zero.

Table 6 Panel - Bootstrapped SAR Coefficients

	Krayot	South	Dan	Tel Aviv	Sharon	Center	Haifa	North	Jerusalem
β	1.074	0.812	0.978	1.109	1.036	0.890	0.419	0.521	0.824
Sd	0.054	0.047	0.033	0.034	0.038	0.073	0.066	0.253	0.043

3.3 Principal Components Estimates

We use equation (5) to test the statistical significance of the eigenvalues of the covariance matrix. Since $N = 9$ in our example we test the hypothesis that the p smallest eigenvalues of the covariance matrix V are equal to the $N-p-1$ 'th eigenvalue. If the null hypothesis cannot be rejected we determine the rank of V to be $k = N-p-1$. When $d = 0$ we find that the rank of the covariance matrix is 9 (full rank). However, when $d = 1$, $k = 4$ according Schott's test. The first four principal components explain x percent of the variance of the first difference of regional house prices.

Using these first four principal components we compute V^* from which we derive the following nonparametric estimates of the W matrix and the SAR coefficients.

**Table 7 Reduced Rank Spatial Connectivity Matrix
(d = 1)**

	South	Dan	Tel Aviv	Sharon	Center	Haifa	North	Jerusalem
Krayot	0.274	0.144	-0.088	0.195	0.398	0.573	-0.159	-0.337
South		0.071	-0.002	0.245	0.531	0.192	0.175	-0.487
Dan			0.214	0.110	0.255	0.127	-0.031	0.153
Tel Aviv				0.120	0.243	-0.053	-0.002	0.566
Sharon					0.155	0.111	-0.028	0.255
Center						-0.421	0.043	0.081
Haifa							0.197	0.304
North								0.065

**Table 8 Reduced Rank Estimates of SAR Coefficients and $\sigma(\epsilon)$
(d = 1)**

	Krayot	South	Dan	Tel Aviv	Sharon	Center	Haifa	North	Jerusalem
SAR	0.790	0.788	0.986	1.109	1.162	1.106	0.668	0.136	0.727
$\sigma(\epsilon)$	0.029	0.001	0.001	0.001	0.001	0.003	0.109	0.356	0.006

Table 7 should be compared with Table 3 for the case in which $d = 1$. Table 8 should be compared with Table 4 for the case in which $d = 1$. The reduced rank estimate of W has a wider range than its counterpart in Table 3. The elements in Table 6 range from -0.49 to 0.57. Connectivity is strongest and positive between Tel Aviv and Jerusalem and it is most negative between Jerusalem and the South. On the whole, the elements in Table 7 are quite different to their counterparts in Table 3. However, Table 8 indicates that the SAR coefficients are similar to their counterparts in Table 3, and the goodness-of-fit in Table 8 is clearly better. The reduced rank SAR models are especially accurate in the South, Dan, Tel Aviv, Sharon and Center. The model continues to fit poorly in the North.

3.3 Asymmetric Estimates

In Table 9 we report the Gini correlation matrix for the log first differences ($d = 1$) of regional house prices. The Gini correlations range between 0.11 and 0.93 whereas in Table 1 their Pearson counterparts range from 0.32 to 0.94. In most cases the Gini correlations are not symmetrical. For example, the Gini correlation between Krayot and the South is 0.632 while between the South and Krayot it is 0.557. In some cases, e.g. Krayot and North the Gini correlations are highly asymmetrical whereas in other cases the correlation are almost symmetrical, e.g. between Tel Aviv and Dan. Table 9 suggests

that imposing symmetry distorts the correlations between regional house prices because the marginal distributions of regional house prices are not "exchangeable".

**Table 9 Gini Correlation Matrix for Regional House Prices
(d = 1)**

	Krayot	South	Dan	Tel Aviv	Sharon	Center	Haifa	North	Jerusalem
Krayot	1	0.632	0.828	0.671	0.769	0.715	0.830	0.536	0.672
South	0.557	1	0.690	0.603	0.693	0.743	0.611	0.696	0.628
Dan	0.835	0.702	1	0.937	0.905	0.890	0.818	0.453	0.912
Tel Aviv	0.700	0.602	0.927	1	0.845	0.820	0.685	0.419	0.929
Sharon	0.801	0.695	0.905	0.851	1	0.805	0.800	0.452	0.887
Center	0.647	0.708	0.836	0.778	0.769	1	0.601	0.516	0.713
Haifa	0.763	0.656	0.760	0.574	0.744	0.564	1	0.447	0.693
North	0.114	0.804	0.474	0.420	0.398	0.619	0.400	1	0.342
Jerusalem	0.644	0.553	0.869	0.933	0.853	0.716	0.667	0.317	1

In Table 10 we report the estimated spatial connectivity matrix, which is asymmetric by design. The elements of W range between -1.6 and 2.8 which is much broader than the range in Table 3. This suggests that imposing symmetry moderates the variation in the estimates of spatial connectivity. The degree of asymmetry is pronounced, and in many cases the spatial effect of region A on region B has the opposite sign of that of region B on region A. Or if the sign does not change, the size of the effect is quite different. For example, the spatial effect of Haifa on Krayot is much larger than the effect of Krayot on Haifa. The spatial effect of the South on Krayot is negative while the effect of Krayot on the South is positive. These results may be compared with Table 3 (d = 1) which were obtained from Pearson rather than Gini moments.

**Table 10 Asymmetric Spatial Connectivity Matrix
(d = 1)**

	Krayot	South	Dan	Tel Aviv	Sharon	Center	Haifa	North	Jerusalem
Krayot		-1.217	2.777	-0.517	-1.287	-1.617	0.977	0.203	1.680
South	0.255		0.314	-0.110	0.381	0.158	-0.101	0.232	-1.130
Dan	0.158	-0.079		0.447	0.041	0.151	0.132	0.048	0.102

Tel Aviv	-0.046	0.004	1.011		-0.042	-0.113	-0.219	0.027	0.370
Sharon	0.224	0.146	0.204	0.332		0.077	0.028	-0.048	0.036
Center	0.141	0.269	0.306	0.609	0.358		-0.153	0.042	-0.573
Haifa	0.370	-0.170	0.934	-0.692	0.076	-0.444		0.277	0.649
North	-1.332	0.919	0.823	0.320	0.167	0.347	0.084		-0.328
Jerusalem	-0.668	0.272	1.327	0.343	0.149	-0.219	0.080	-0.283	

In Table 11 we report the Gini estimates of the SAR coefficient, which may be compared with their Pearson counterparts in Table 3 ($d = 1$). The SAR coefficients are relatively large, and have the same order of magnitude as in Table 4. However, individual SAR coefficients are quite different. For example in Table 4 the SAR coefficient for the North is 0.262, while in Table 11 it is 1.451. Indeed, Table 11 suggests that there is a spatial unit root in the first difference of regional house prices.

Table 11 Gini Estimates of SAR Coefficients and $\sigma(\varepsilon)$
($d = 1$)

	Krayot	South	Dan	Tel Aviv	Sharon	Center	Haifa	North	Jerusalem
SAR	0.553	1.120	1.044	1.034	1.172	1.198	0.905	1.451	0.748
$\sigma(\varepsilon)$	1.267	0.645	0.198	0.269	0.492	0.604	0.585	1.321	0.370

Finally, Table 11 shows that the goodness-of-fit of the Gini estimates of the SAR model is ubiquitously poor.

3.4 Testing Restrictions using GMM

Figure 1 Regional House Prices in Israel
Logs at 1991 prices

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Map 1: Geographic Regions of Israel

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